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QUESTIONS OF EARLY SCALING[†] IN INCLUSIVE REACTIONS

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I. SCALING AND ITS VARIOUS FACETS

One of the most interesting features of the many inclusive distributions[1] which are being extensively studied at present is their possible scaling property. An inclusive distribution depends on three variables which are generally chosen as the center of mass energy squared s, the longitudinal center of mass momentum of the observed secondary q_L and its transverse momentum squared q_T^2. Instead of q_L one also often uses the rapidity y defined as $y = \mathrm{Log}\,\dfrac{q_L + W}{\sqrt{q_T^2 + m^2}}$, where W and m respectively stand for the center of mass energy and mass of the secondary. The inclusive distribution is then defined in a Lorentz invariant way as:

$$f = \omega \frac{d\sigma}{dq_L\, dq_T^2} = \frac{d\sigma}{dy\, dq_T^2} \qquad (1)$$

A function of three variables it may eventually depend only on two at sufficiently high energy, the longitudinal momentum dependence scaling according to the total center of mass energy $\sqrt{s}$. The inclusive distribution may then be considered as a function of q_T^2 and $x = 2q_L\sqrt{s}$ only. Such a scaling property is a prominent feature of some theoretical models. It has been strongly advocated by Yang and his coworkers[2] and by Feynman[3]. An asymptotic property it is predicted to apply at sufficiently high energy.[4]

As shown by Mueller[5] the corresponding limiting behavior can also be defined in the framework of Regge Theory. The inclusive distribution[1] defined for a reaction AB(C)[6] is then identified with an $s_{AB\overline{C}}$ discontinuity of the ABC elastic forward amplitude.[7] Its asymptotic limit is defined by asymptotic Regge behavior corresponding to Pomeranchon exchange. If the rapidity difference between particle A and C (B and C) is large enough, a limiting behavior depending on q_T^2 and x is reached. This can be identified with a limiting fragmentation of particle A (B). Particle C, a fragment of the projectile A (target B) then shows a small rapidity difference with A (B) or has only a small energy as seen in the rest frame of particle A (B). The inclusive distribution asymptotically scales and is expected to do so the faster the smaller the rapidity interval between C and A (B) is or, as a result, the larger the rapidity interval between C and B (A) can be, thus justifying the corresponding Pomeranchon exchange approximation.[8]

If the energy is high enough, the rapidity difference between C and both A and B can be large, thus justifying a Pomeranchon exchange approximation in two variables. The limiting inclusive distribution is then found to depend on q_T^2 but no longer on the rapidity of particle C. One reaches the pionization limit advocated by Feynman.[3] This is a new type of asymptotic scaling first expected to be found for secondaries with small centers of mass momentum. The asymptotic distribution should exhibit a plateau in the central rapidity region limited by edges with fixed shapes corresponding to the limiting fragmentation of the target and projectile (Fig. 1).

This very general approach may be considered as an extension of the one followed for total cross section which, through the optical theorem, are related to the absorptive part of the elastic forward amplitude or to its s_{AB} discontinuity. A Regge approximation of this discontinuity gives an asymptotic limit (Pomeranchon exchange) corresponding to a constant total cross section.

The $s_{AB\bar{C}}$ discontinuity relevant for an inclusive distribution presents some further complications. One has to specify the limit prescribed for the s_{AB} variable[5, 9, 10] since the $s_{AB\bar{C}}$ discontinuity has an s_{AB} discontinuity. The required choice is presented in Fig. 2a. In any case, a Regge approximation, justified for large enough $s_{AB\bar{C}}$, yields asymptotic contributions (Pomeranchon exchange) which can be envisaged when one rapidity interval AC (BC) is large enough or when both the AC and BC rapidity intervals are large. They are represented graphically on Fig. 2b and 2c respectively. The first one corresponds to target fragmentation (C is considered as a fragment of B). The second one corresponds to the pionization limit with a uniform rapidity distribution for particle C .

The same general theoretical framework thus leads to constant asymptotic cross sections and to scaled inclusive distributions.[5] Nevertheless, if the scaling limit is thus defined and if scaling appears as a very general property, the approach to the scaling limit is not yet specified.

For total cross section the approach to the asymptotic limit is controlled by secondary trajectories which typically contribute terms decreasing as $s^{-1/2}$. The s_{AB} dependence of the cross section is then approximated as

$$\sigma_{tot}^{AB} \simeq \sigma^{AB} + r^{AB}\, s^{\alpha(0)-1} \qquad (2-a)$$

where σ^{AB} is the limiting cross section and $\alpha(0)$ stands for the intercept of the leading secondary trajectory. The same type of formula can be tried for the $AB\bar{C}$ discontinuity attached to an inclusive distribution. In the fragmentation region of particle A (or B) one thus writes:

$$\left(\omega \frac{d\sigma}{dq_L\, dq_T^2}\right)_{BC}^{A} \simeq f_{BC}^{A}(x, q_T^2) + g_{BC}^{A}(x, q_T^2)\, s^{\alpha(0)-1} \qquad (2-b)$$

where $\alpha(0)$ is now the intercept of the leading secondary trajectory which can be exchanged in the $AB\bar{C}$ channel. Equation 2b is written after the graph of figure 2b. Only one rapidity interval (AC) is then large, the other one (BC) may be small. The same of course applies in the projectile fragmentation region with of course different functions f and g.

Even though nothing yet prescribes the energy scale against which the approach to the limiting behavior should be most conveniently measured, one is lead to expect a relatively rapid (power law) behavior. The non limiting contributions to the inclusive distribution could decrease at least as $s^{-1/2}$ in much the same way as the non constant part of the total cross section. The same phenomenology could also apply.[11]

Such a behavior can already be tested at accelerator energies where total cross sections show such a Regge behavior[12] and where one of the rapidity gap AC (BC) can be chosen almost as large as the AB one. Testing the approach to pionization which could now involve an $s^{-1/4}$ term instead of $s^{-1/2}$[5, 13] may require at least ISR energies. Our discussion of the approach to scaling will be essentially limited to the fragmentation region where relation (2b) would apply.

At 20 GeV several cross sections have already almost reached what seem to be their asymptotic limit. One may, therefore, expect that the same applies to inclusive distributions or at least to some of them. Furthermore, some total cross sections are remarkably constant and this from relatively low energy. This is in particular the case for the K^+p and pp total cross sections and, according to duality,[14] it is expected to hold whenever the reaction channel has exotic quantum numbers. There is then no resonance contribution to the s_{AB} discontinuity and it gets contributions only from a "featureless" diffractive background.[12] One may then expect that some inclusive distributions could show an early scaling property, that is scaling at 10 GeV already! However, defining a proper criterion which would generalize the Harari-Freund rule for total cross sections apprears as a rather difficult task.[9, 11, 15, 16] We shall come back to it in section 2. Several rules which have been proposed so far are summarized in Table I. Almost any possibility is open!

We may at this stage turn to experiment to see what evidence is already available. Figure 3, taken from Ref. 17 shows data on $pp(\pi^{\pm})$ at 28.5 GeV/c[17] and at several ISR energies.[18] At $q_T^2 = 0.16\ (\mathrm{GeV/c})^2$ all collected results beautifully supports scaling. The data correspond there to x values which clearly refer to the proton fragmentation region $(0.1 < x < 0.4)$[19]. On the other hand the scaling limit, if any may not yet be reached at 30 GeV for the $q_T^2 = 0.04\ (\mathrm{GeV/c})^2$ results which correspond to relatively low values of x. If one first focusses on the obvious proton fragmentation region ($0.1 < x < 0.4$ say), the scaling property is also demonstrated experimentally for $pp(\pi^-)$.[20] The π^+/π^- ratio at ISR energies agrees with the one measured at present machine energy.[17] It seems to reach 1 at low x values, as indeed expected for pionization pions[3] when 30 GeV data give a higher value at $x = 0$. This again indicates that the scaling limit is probably not yet reached at 30 GeV/c when one considers the neighborhood of $x = 0$ even if it is beautifully met already in the typical fragmentation region. This is shown on Fig. 4. Also shown is the $\bar{p}/\pi^-$ ratio which is found to be (as naively expected) much higher at ISR energies than at standard machine energies. The $pp(\bar{p})$ inclusive distribution does not scale early if the $pp(\pi^-)$ does!

If scaling properties are most dramatically exposed when jumping from 30 to 1500 GeV, as possible at present with ISR results, they can also be analyzed within the relatively limited range of energy offered by accelerators. This is illustrated by Fig. 5a which shows the 8 and 16

GeV/c $\pi^+ p(\pi^-)$ distributions.[21] In the proton fragmentation region ($x < -0.1$) the two distributions are compatible within statistical errors. This is an early scaling, and, as discussed in detail later, it should be an actual one. The inclusive distribution also scales in the pion fragmentation region ($x > 0.1$) except for peaks at large x values which are easily traced to resonance quasi two body formation and should deserve a special study anyway. However, at low $|x|$ values, the measured distribution does not scale. It increases with energy, a trend which may continue to very high energy as suggested by the comparison between 30 GeV and ISR results shown on Fig. 3.

The very early scaling of the π^- distribution in the proton fragmentation region may be contrasted with the behavior of the $\pi^+ p(\pi^+)$ distribution in the same x range.[21] It sharply decreases in the proton fragmentation region between 8 and 16 GeV/c. Of course in view of the ISR results on proton fragmentation[21] (Figs. 3 and 4), we should expect different limits. It remains though that the π^- distribution seems to reach its limiting value even more rapidly than the π^+ distribution. Some early scaling property can also be seen in the π^+ fragmentation region proper, at intermediate positive x values. At large x values, the disbribution is dominated by a leading particle effect, the observed π^+ being easily traced to the incident π^+ seen after its quasi elastic scattering off the proton target. This effect strongly differentiates the π^+ and π^- distributions at large pion momentum. It is generally expected to give a peak at large x in the inclusive distributions which first sharpens and moves toward $x = 1$ with increasing energy. In the triple Regge approximation[22] applicable here for $x \approx 1$, this would correspond to a PPR coupling considered as the leading term in the absence of triple Pomeranchon coupling. The quasi elastic peak does not scale. Were the triple Pomeranchon coupling be non vanishing it would however scale eventually.[23]

For the central x region, the π^+ distribution, again observed between 8 and 16 GeV/c manifests a scaling property which is not found in the π^- distribution. The two π^+ distributions are compatible within statistics! When probing for asymptotic scaling properties this effect should, however, be considered as a spurious one. An asymptotic non zero limit for slow center of mass pions should not differentiate between π^+ and π^- when the values of f(0), measured for the $\pi^+p(\pi^+)$ and $\pi^+p(\pi^-)$ respectively, differ by as much as a factor 1.6.[21]

Such a phenomenon is not the scaling property which we may try to see at present machine energy already. It is referred to as <u>Pseudoscaling.</u>[24] Met a relatively low energy, 10-20 GeV, it should disappear with increasing energy, the inclusive distribution around $x = 0$ reaching its eventual limit from below. In particular, it should no longer hold at NAL energies. Pseudoscaling is a property expected from reactions where low yield collisions are not discriminated against. In $\pi^+p(\pi^+)$ for instance all channels contribute and in particular the inelastic two prong channel which still represent a large fraction of the cross section at 20 GeV but which decreases with energy. In $\pi^+p(\pi^-)$ on the other hand, one considers only reactions with four prongs or more. As discussed later a configuration with a fixed (low) prong number is expected to contribute at x = 0 an amount which decreases with energy even if it scales at intermediate $|x|$ values,[24, 25] the bulk of the distribution in the neighborhood of x = 0 being provided by configurations with increasing multiplicities the cross sections of which increase with energy to reach a limiting value[2] or individually decrease after going through a maximum.[3] The apparent stability of the $\pi^+p(\pi^+)$ distributions at x = 0 is then due to a compensation between the decreasing two prong contribution[26] and the increasing four and more prong contributions. The $\pi^+p(\pi^-)$ distribution, with no decreasing two prong component, is on the contrary expected to rise. A similar phenomenon should be observed in the $K^+p(K^o)$ or $K^-p(\overline{K}^o)$ distributions. They should show a pseudoscaling property at present machine energy[27] which should again disappear with increasing energy. At the same time the $K^+p(\pi^-)$ inclusive distributions should rise at x = 0 at present machine energy, with contributions from four and more prong reactions only.

These different facets of scaling are of course connected through sum rules.[28] The separate inclusive distributions (1) written for each particle species i observed in a reaction are constrainted by energy conservation. This reads:

$$\sqrt{s}\ \sigma = \sum_i \int \omega \frac{d\sigma_i}{dq_L dq_T^2}\, dq_T^2\, dq_L \tag{3}$$

or

$$2\sigma = \sum_i f^i(x)\, dx \tag{4}$$

with $f(x) = \int \omega \frac{d\sigma}{dq_L dq_T^2}\, dq_T^2$

If the total cross section reaches relatively quickly and smoothly its limit so does the sum of the integrated inclusive distributions. However, each of them can separately still show some prominent variations. In particular if the total cross section varies very little with energy (pp) when some inclusive distributions decrease, others should increase at least in some range of x in order to compensate.[16] This should be contrasted with the general decreasing Regge behavior of total cross section imposed by duality.[14] It should be stressed though that the leading particle distribution always contributes to (4) when the shape of the quasi elastic peak may as previously discussed change with energy.[29]

Sum rules for internal quantum numbers can also be useful. For instance charge conservation gives:

$$(Q_A + Q_B)\,\sigma = \sum_i \int Q_i \frac{d\sigma_i}{dq_L dq_T^2}\, dq_T^2\, dq_L \tag{5}$$

If the π^+ and π^- inclusive distributions scale in pp collisions as we know they do (Fig. 4), the π^+ distribution should be on the average above the π^- one. Their reaching the same limit at $x = 0$ is thus connected to their keeping their low energy distance at intermediate x values.

Scaling, therefore, takes many different aspects and the range in x should be specified as well as the particle species. Whether or not an asymptotic property,[30] it appears to be a prominent fact for pions in what can be unambiguously labelled as the fragmentation region ($|x| > 0.1$ say) and interests an already impressive energy range.[17] Furthermore, in some reactions ($\pi^+p(\pi^-)$ for instance) it appears extremely early. In the central region, however, (neighborhood of $x = 0$), any early scaling already met seems to be of a spurious kind. Inclusive distributions seem to rise there between present machine energy and ISR energy. Even if they further rise within the ISR energy range,[30,31] a scaling limit may eventually be reached. Double exchange (Pomeranchon, Reggeon) terms in Fig. 1c give a non scaling term[13] which behaves as $s^{-1/4}$, together with the scaling double Pomeranchon exchange contribution. This is still compatible with present data as shown on Fig. 6. In any case scaling in the low rapidity region appears as a more remote property than what we may already ascertain in the fragmentation region. This latter property became a working hypothesis through cosmic ray studies[32] and can now be analyzed in details combining 10-30 GeV results with ISR data and soon information from NAL.

This is what we will focuss upon here.

Yet a new type of scaling should be eventually met in the fragmentation region. Inclusive distributions should scale according to total cross sections. This is readily seen from Fig. 2b in the Pomeranchon exchange limit. For example in the proton target fragmentation region the inclusive distribution obtained from pion and Kaon projectile should be in the ratio of the Pomeranchon couplings which is equal to the ratio of the πp and Kp total cross section. Such a property is reasonably well met by data. See Ref. 21 (T. Ludlam) for a recent review.

II. EARLY SCALING AND DUALITY

The early limiting behavior of some total cross section is connected to the exotic or non exotic character of the reaction channel. The Harari-Freund rule[14] operates through a decomposition of the imaginary part of the forward elastic amplitude (s_{AB} discontinuity) into a constant diffractive part (always present) and a decreasing Regge contribution present only if there are s channel resonances. Their positive contributions to the imaginary part of an elastic amplitude are dual to positive decreasing Regge terms.[33] An inclusive distribution $f^{A}_{BC}(x, q_T^2)$ being proportional to an $AB\bar{C}$ discontinuity of the ABC elastic forward amplitude, a simple generalization may be tried, postulating no Regge contribution, and hence scaling, if the $AB\bar{C}$ channel has exotic quantum numbers.[11]

This is to be applied in the fragmentation regions (Fig. 2b) when s_{AB} and $s_{A\bar{C}}$ (or $s_{B\bar{C}}$) are both large. A simple Regge and dual behavior may then apply in the $AB\bar{C}$ channel when calculating the discontinuity. Such a criterium meets success when confronted with the recent results on $\pi^+ p(\pi^{\mp})$.[21] In the proton fragmentation region the first distribution scales very early ($AB\bar{C}$ exotic) when the second one does not scale between 8 and 16 GeV/c ($AB\bar{C}$ non exotic). However, we already know looking at the behavior of $pp(\bar{p})$, ($AB\bar{C}$ exotic) that the rule a priori have but a limited validity. It may nevertheless be useful. Two remarks are in order. First the data (slower scaling for π^+ fragmentation) may be interpreted on more naive grounds. Pion charge exchange (with a decreasing cross section) will give Δ^{++} which contribute to the π^+ but not to the π^- distribution. The slower π^+ scaling is then connected to the relatively marked decrease of inelastic two prong cross sections such as $\Delta\pi^o$ which are not relevant for the π^- distribution. If this is the case one would predict the $K^+p(\pi^-)$ distribution to scale more rapidly than the $K^+p(\pi^+)$, when they should both show an early scaling according to the exoticity rule.[11]

Such a statement, even if backed experimentally, does not belittle the general Regge analysis of Mueller applied in this case. It suggests, however, that the energy is not high enough for its simple application and that other mechanisms have first to become of less importance.

The same argument could be made for the pp(π^-) reaction as compared to the pp(π^+) one. Detailed experimental information would be very interesting. At the same time π+p(K^-) should show . an early scaling since $AB\overline{C}$ is exotic when more naively speaking, one would expect that only at verly large energy can the proton or the π^+ easily fragment into a K^-, together with an other K as well as pions! Again the rule may be useful but does not contain an energy scale which could be much different for πp(π) on the one hand and πp($\overline{K}$) on the other hand!

This may be understood if one looks at the Pomeranchon exchange approximation in Fig. 2b as the shadow of the many pions produced on a multiperipheral chain when the AC (or BC) rapidity interval is large. If C is the $\overline{K}$ fragment of a proton (particle B) one is lead to expect <u>other</u> proton fragments (at least a K and a proton and in general some pions). As a result few secondary will be expected in the rapidity interval limited by the rapidities of A and C respectively and consequently the asymptotic Regge approximation could still be very poor at 10 or 20 GeV/c. We would then expect scaling with a predictable hierarchy: first pp(π^-), then pp(K^-), then only pp($\bar{p}$). Detailed information on such distributions (at $0.1 < x < 0.4$ say) should be very interesting.

The second point refers to the neccessary rather than sufficient characters of the condition. It can be shown that in the framework of the Regge pole approximation, which includes factorization, a condition on $AB\bar{C}$ alone leads to some problems. They can be removed if one requires that not only $AB\bar{C}$ but also AB has exotic quantum numbers.[9] This does not remove though the problems with $pp(\bar{p})$ or $pp(K^-)$[34, 35] discussed above. It, however, tackles the fact that the relevant $AB\bar{C}$ discontinuity has to be taken when s_{AB} is separately approached from above and below in the two blobs of the discontinuity equation (Fig. 2a). If resonances are present in the AB channel, one expects that the $AB\bar{C}$ discontinuity, which has an AB discontinuity, should manifest through duality a corresponding Regge behavior.[36] From a pragmatic point of view both criteria should best be differentiated in a pion inclusive reaction for which the preceeding difficulties are not met. With $AB\bar{C}$ exotic only, one expects that $K^-n(\pi^+)$ should show an early scaling when $K^-p(\pi^+)$ should not. With the AB exotic condition, both should not show any early scaling. Through casual guess both distributions should look alike at least in the Kaon fragmentation region. Again experimental information would be very interesting. Duality may give stringent constraints!

Calculating the relevant $AB\bar{C}$ discontinuity involves not only the imaginary part of the $AB \to C + x$ amplitudes for which a resonance approximation is suggested by duality but also their real part for which duality in the large has nothing to say. Resonances in the $B\bar{C}$ or $A\bar{C}$ channels will in general include Regge variations of the Real part at high energy which may not average to zero. This leads to still other criteria.[15] At present any definite answer requires a specific model and dual models appear as providing an interesting guiding line. They do not constrain however possible subtraction terms giving important contributions to the real part of production amplitudes. The different conditions so far proposed are summarized in Table I.

Before we turn to it we may mention a specific configuration for which agreement prevails that $AB\bar{C}$ exotic is a necessary and sufficient condition. This is the triple Regge limit[22] relevant when both s and the missing mass M to be inclusive particle C are large, with the further condition s/M^2 large. Particle C is then obtained from particle A through Reggeon exchange (s/M^2 large) and the Reggeon B amplitude (at large M) is approximated in terms of Regge contributions (Fig. 7). The inclusive cross section is proportional to the absorptive part of a Reggeon particle amplitude and, if the $AB\bar{C}$ channel is exotic, we should find only a Pomeranchon contribution which behaves as M^2. The inclusive distribution of particle C then scales and already for relatively low missing mass. As an example we expect the reaction $K^+p(\pi^+)$ (K^* exchange) to scale early when pp(n) (pion exchange) should however not scale at low missing masses. Reaction pp(Λ) should again scale at low missing masses. One may select either a very fast or very slow Λ (small transfer to either proton). All this supposes that the Reggeon Reggeon Pomeranchon coupling does not vanish. With no triple Pomeranchon coupling Pomeranchon exchange in the $A\bar{C}$ channel does not contribute to the scaling limit.

III. SCALING IN A DUAL MODEL

One may write a six point dual term for the elastic ABC amplitude and study its $AB\bar{C}$ discontinuity. A prominant feature is the transverse momentum cut off (exponential in q_T^2) which it exhibits in contradistinction with the amplitude proper. One is thus lead to separate seven different components in the inclusive distribution,[37] some of which only yield a non vanishing scaling limit. We prefer here to start from a dual model for production amplitudes which generalizes the dual scheme for scattering amplitudes recently proposed by Veneziano.[38] The three basic elastic terms are shown on Fig. 8a. The first term V has poles in the s channel, the second one B has poles in both the t and u channels but no pole in the s channel, the third one D has no pole but a cut in s and only vacuum quantum numbers in the t channel. Both V and B are standard tree dual terms when D stands for Pomeranchon exchange as it most simply appears in a dual model. As advocated by Veneziano these three terms may be used as input in a calculation scheme giving a model amplitude respectful enough though not fully of both crossing symmetry and unitarity. To these three terms one may associate three production amplitudes, letting the produced Reggeon(s) split among the observed secondaries (Fig. 8b). Isolating each time one of the secondary and summing over all the other ones, several different contributions to the $AB\bar{C}$ discontinuity of a six point dual term can be reached.

In order to illustrate how the procedure goes, we start with the V term taking the observed secondary C in any of its three topologically different locations uppermost, lowermost and intermediary. This gives three different terms which are component 1, 2 and 4 in the notations of Ref. 37. The corresponding diagrams of the six point dual term are displayed on Fig. 9a. They obviously all give vanishing contribution asymptotically (resonances in the AB channel). They respectively require AB, $B\bar{C}$ and $AB\bar{C}$ non exotic; AB, $A\bar{C}$ and $AB\bar{C}$ non exotic and AB non exotic with vacuum quantum number in $C\bar{C}$. If AB is non exotic, the non scaling component 4 will contribute a Regge term. Further conditions are, however, necessary in order to forbid all Regge terms. Starting with the B term, we may take the observed secondary as the only particle produced at either end. Summing over all other external lines and untwisting both sides one obtains a tree graph with ACB ordering ($A\bar{C}$, $B\bar{C}$ and $AB\bar{C}$ must now be non exotic). This is component 3 of Ref. 37. It again has a vanishing scaling limit. More interesting terms are reached when C is taken as one of the several secondaries attached to the two Reggeons in the B term. There are six a priori different topological locations but one gets only three topologically different dual terms. This construction neglects crossed terms which are considered in Ref. 15. If C has an exterme location, one either gets component 5 ($B\bar{C}$ non exotic and vacuum quantum numbers in $A\bar{A}$) or component 6 ($A\bar{C}$ non exotic and vacuum quantum numbers in $B\bar{B}$). Both terms have a scaling and a non scaling piece! Component 5 eventually scales in the target fragmentation region whereas Component 6 scales in the projectile fragmentation region. The asymptotic scaling of Component 5 say results from the exchange of the quantum numbers of the vacuum in the $A\bar{A}$ channel when a large rapidity interval exists between A and C (B fragmentation). An important point is that, as expected from wisdom gained with the multiperipheral model, one does not need Pomeranchon exchange in the basic production term in order to reach a scaling inclusive limit. As an example, the amplitudes leading to Component 5, together with the corresponding dual term are shown on Fig. 9b, the connection is obvious. If C now has an intermediate location among the secondaries of either Reggeon, one gets a scaling terms with vacuum quantum numbers in all $A\bar{A}$, $B\bar{B}$ and $C\bar{C}$ channels. This is the seventh component of Ref. 37. One may proceed further and define separate components for the D terms and the mixed BD terms.[39]

The main interest in itemizing such dual terms with their respective quantum numbers constraints is to display different non scaling term but <u>mainly</u> different scaling terms. Using sum rules (4) and conditions on σ (AB exotic or not) one also reaches some prediction about the sign of the non scaling contributions.[16] As an application of the preceeding decomposition we may consider $\pi^{\pm}p(\pi^{-})$ where $AB\bar{C}$ and $A\bar{C}$ are exotic in the first case (component 4, 5 and 7 only) whereas no channel has exotic quantum numbers in the second case (all components). They are, therefore, expected to differ asymptotically in the projectile fragmentation region. This is due to the presence of the scaling part of the 6th component in the second distribution only. Since component 5 (as well as component 7) has vacuum quantum numbers in $A\bar{A}$, we expect at the same time the same scaled contribution in the proton fragmentation region. Both features are verified experimentally. This is a success but as already discussed these different limits also follow from simple fragmentation properties.[24]

Such a detailed dual model approach leads to conclude that all channels should be exotic in order to exclude any Regge ($s^{-1/2}$) behavior in the inclusive distribution.[16] No decisive prediction is yet reached, however, about the approach to scaling and if the seventh component, relevant for $pp(\bar{p})$ or $pp(K^-)$, has only a scaling part, we know that it is not enough in order to meet experiment.[20] One would say, however, that there is a belated onset (due to obvious kinematical factors) of a quick scaling. One would predict the $pp(\bar{p})$ distribution to almost jump to its limiting value. Results in the ISR and NAL energy range will be extremely useful. Some recent data are presented on Fig. 10. They indeed suggest a sharp rise followed by a plateau as if no Regge term would exist beyond the belated increase of the cross section corresponding to threshold effects.

Duality can be used to predict the presence or absence of Regge terms relevant beyond such threshold effects and dual models show that crossed channel ($A\bar{C}$ and $B\bar{C}$) are a priori important together with the direct ones (AB and $AB\bar{C}$). At present, however, the most detailed dual approach does not yield to a definite hierarchy of scaling properties attached to the presence of non exotic quantum numbers in the different channels. We are left with very strong requirements and exotic quantum numbers in one key channel is obviously not enough to enforce an early scaling property.

We may hope for some progress which could be prompted by more experimental information. We see, however, that threshold effects attached to the type of secondary are extremely important at present machine energy and in effect the key ones in some reactions such as $pp(\bar{p})$. They have to be analyzed in a different way. They obviously result from the fragmentation properties of the beam or target particle into relatively heavy final states. This is the question to which we turn now.

IV. EARLY SCALING IN A FRAGMENTATION MODEL

We have seen that if an early scaling is to be found in some reactions it appears in the fragmentation region. It is then natural to look at the observed secondary as part of a cluster obtained through the excitation of either the target or the projectile. The x dependence of the inclusive distribution indicates the effective excitation mass involved. We may assume an isotropic distribution in the cluster rest frame with both cluster and inclusive secondary sharing the same rapidity. At high energy ($M \ll \sqrt{s}$) an excitation mass M of 2 GeV will then give a pion distribution with maximum at $x = 0.15$ (transverse mass of 300 MeV). The shape of the distribution is controlled by the contribution of relatively light clusters[24] whereas the central part of the x distribution is mainly obtained from the heavier ones.[40] Such hadronic clusters obtained when an incident particle comes back to its ground state after emitting a bright flare of pions are referred to as Novae.[24] An exponential distribution in q^2 in the nova rest frame, which reproduces the observed q_T^2 distribution, is then found to give a Gaussian scaled inclusive distribution at larger x. For each fixed nova mass M: $f(x) \simeq \exp(-2x^2M^2/\langle q_T^2\rangle)$.[24] For values of x where the secondary pion moves in the opposite direction to that of the nova it originates from, and in particular for $x < 0$, the inclusive distribution decreases exponentially with s at least when $s \gg M$.

Due to the loose but manifest connection between each nova mass and a relevant x domain, low mass novas ($M \approx 2$ GeV) contribute at larger x values ($0.1 < x < 0.3$ in $pp(\pi)$) while the heavier ones should not much affect inclusive distribution beyond $x = 0.1$.[41] The fragmentation model thus devised does readily lead to scaling but in a very selective way. If the nova mass spectrum becomes eventually independent of the incident energy, scaling will hold. For each nova mass, vacuum quantum number exchange should eventually win over other exchange mechanism with increasing energy and one expects an asymptotic scaling property with Regge behaved non scaling contributions differentiating between internal quantum numbers. This should be best discussed according to the general Mueller-Regge picture applied to the approach to limiting fragmentation . At the same time one should use the nova picture, neglecting quantum number exchange, in order to readily analyze the important threshold effects and the expected differences between scaling limits. As an example one expects that the $\pi^- p(\pi^-)$ distribution should be found above the $\pi^+ p(\pi^-)$ in the proton fragmentation region $(x < -0.2)$[41] but that both distributions should tend toward each other with increasing energy. The observed difference is attributed to charge exchange contributions which for fixed nova mass should decrease with energy. In the same region the $\pi^- p(\pi^+)$ distribution is, however, predicted to reach a limit which is higher than the one obtained for $\pi^- p(\pi^-)$ since the low mass proton novas, which mainly contribute to it, yield more π^+'s than π^-'s. Quantitative estimates are in very good agreement with experiment.[(24)] For the very same reasons, the $\pi^- p(\pi^-)$ distribution in the pion fragmentation region $(0.1 < x < 0.4)$ will scale at a higher value than the $\pi^- p(\pi^+)$ distribution (about a factor 2 higher).

The approach to the very general limiting fragmentation limit can thus be made quantitative. A relatively large fraction of the inelastic cross section is attached to the excitation of low mass novae already possible at relatively low energy ($M < 3$ GeV say). The approach to scaling now mainly controlled by quantum number exchange in "quasi two body" reactions is expected to be fast. Increasing the energy permits higher mass excitation but they do not, however, affect much the shape of the x distribution for pions since they mainly contribute around $x = 0$. As shown on Fig. 4, the π^+/π^- ratio at $|x| > 0.1$ does not appreciably depend on the incident energy.

Now comes the important point. Conditions sufficient for inclusive pions may well not be sufficient for other secondaries. It takes a heavier nova to yield a K^-. It takes a higher excitation mass still to produce a baryon antibaryon pair. One, therefore, gets an easy set of rules with respect to scaling and a definite hierarchy for scaling in the fragmentation region, pion inclusive reactions being the first to scale by far, then followed by K^- (with non strange initial particles) and then by $\bar{p}$. No contradiction is met so far and a large amount of predictions can be made in a straight forward way.[42] As an example among many, the $K^+p(K^0)$ distribution should scale rather fast from below in the proton fragmentation region ($x < -0.3$, very small cross section). It should, at the same time scale relatively quickly in the Kaon fragmentation region ($x > 0.1$). Around $x = 0$ it should pseudoscale, the increasing contributions from heavier novas compensating on an extended energy range (5 - 20 GeV say) the decrease of the contributions of the lighter ones (and in particular K^*).

Rephrasing scaling properties in a specific model may appear as a step backward from the very general Mueller Analysis and its dual discussion. What is gained though is an actual energy scale for the setting of the expected scaling properties as a function of particle species <u>and</u> x values. Such effects are often more important than those attached to quantum number exchange. Scaling properties also eventually apply[45] for reactions where one specifies several final particles ($K^+p(\pi^-)$ with 3 π^- seen, say). This readily also holds in fragmentation models. However, since such distributions get relatively more contributions from heavier novae than those for which nothing is imposed, the observed distributions should be narrower and scaling should be slower,[24] and the more so the longer the list of the required secondaries is! In special cases where it is extremely unlikely that two particles of the same selected kind are produced in any single event ($K^+p(\Lambda, n\pi^-)$ say), the Λ distribution for $\Lambda n\pi^-$ events should normalize to the corresponding cross section σ_n which is practically the same as the corresponding integrated rate. If it is obvious that, in the large, the inclusive Λ distribution will scale according to the different measured cross sections σ_n, scaling should now be verified on the actual shapes of the distributions. It should not come early if n is large.

The many statements which can be made about the fragmentation region are not obviously relevant at low x values where most of the contributions come from the heaviest novae which become accessible as the incident energy increases. A fragmentation picture apparently successful at lower energies may even be of little use when applied to very large excitation masses. Its relevance or lack of relevance there should be best probed through measurements of correlation coefficients as a function of energy.[44] In any case it predicts a slow approach from below at $x = 0$ which is not in contradiction with information available at present. The large excitation masses are expected to yield as many π^+ as π^- and the π^+/π^- ratio at $x = 0$ should as a result approach 1 with energy (decrease to one in pp collisions, rise to one in π^-p collisions). If there is a scaled $x = 0$ limit its approach seems to be very slow in contradistinction with the early scaling already met in the fragmentation region, at least in some reactions, with an understood hierarchy according to the selected secondary. We now briefly turn to the central region about which information starts coming from the ISR.

V. SCALING IN THE CENTRAL REGION

If fragmentation models are apparently successful at describing the shape of inclusive distributions (larger rapidity behavior) observed at present machine energy, the low rapidity pion secondaries, which exhibit a flat distribution at ISR energy, should perhaps be described in a different way.[3] Fragmentation models, useful in discussing the approach to scaling, meet the whole inclusive distribution and predict a flat rapidity which results from the required logarithmic increase of the multiplicity.[24] This is, however, an asymptotic feature which is more readily expected in multiperipheral models and one may see in the flat ISR rapidity distribution ($|y| < 1$), the result of a production mechanism not already present at lower energies (10 - 20 GeV). If almost all models now include a logarithmic increase of the multiplicity and linked to it a central plateau in the rapidity distribution, they drastically differ when it comes to correlations among secondaries. In fragmentation models observation of a secondary (π^- say) at rapidity y_1, implies that a second π^- is likely to be seen at a similar rapidity y_2 (single excitation) or at an opposite rapidity (double cluster excitation). This imposes strong long range correlations. In practice, however, correlations already measured at present machine energy show strong maxima at $y_1 \approx y_2 \approx 0$, the fact of the matter being that most secondaries are produced with low rapidity.[44] Measuring correlations as a function of multiplicity would help refine our understanding of cluster formation and fragmentation at present machine energy but extended to ISR energies such a mechanism would definitely predict strong positive correlations at low rapidity. A low rapidity pion is likely to originate from a heavy slow moving cluster yielding many pions all at low rapidity. On the other hand, a multiperipheral picture predicts a relatively uniform rapidity distribution for each collision (this is how it may be defined) and the observation of one π^- at low rapidity does not provide a strong constraint on the rapidity distribution of its fellow π^-'s. Correlations should of course exist and are expected to decrease exponentially with rapidity interval in a multi Regge model[45] but they should not be as strong. They should eventually scale or depend on the rapidity difference y_1-y_2 only instead of depending on both y_1 and y_2 as they do in a fragmentation picture where the observation of the rapidity of a secondary already gives some information about the cluster mass and hence the multiplicity of the particular event (at least the multiplicity of those pions with the same sign of rapidity when two well separate clusters are formed). Whether or not correlations also approach a scaling limit and how early they do so is a challenging question for ISR experiments to come.

ACKNOWLEDGEMENTS

Conversations with Ed Berger, D. Gordon, H. Lipkin, R. Slansky and G. Veneziano are gratefully acknowledged. I am also thankful to R. Arnold for prompting this review.

TABLE I

A long series of letters. C is considered as a fragment of B.

H. M. Chan, et al.	PRL <u>26</u>, 672 (1971)	$AB\bar{C}$ exotic	Regge behavior and Duality
J. Ellis, et al.	PL <u>35</u>B, 227 (1971)	$AB\bar{C}$, AB exotic	Same input but also using factorization
H. M. Chan, et al.	PL <u>36</u>B, 79 (1971)	$AB\bar{C}$ exotic good as a first approximation	When $B\bar{C}$ is non exotic $AB\bar{C}$ exotic is enough if one limits oneself to lowest order duality diagrams. Pomeranchon exchange would be higher order!
M. B. Einhorn, et al.	PL <u>37</u>B, 292 (1971)	AB, $A\bar{C}$ exotic	Consideration of a class of duality diagrams. Special rule for Pomeranchon exchange.
H. Tye, et al.	PL <u>38</u>B, 30 (1971)	All exotic	Planar duality diagrams
M. Kugler, et al.	PL <u>38</u>B, 423 (1972)	$B\bar{C}$ exotic is the condition if any	$AB\bar{C}$ exotic as an input condition, factorization, symmetry.

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[1] Extensive reviews are already available about inclusive distributions: E. L. Berger "Phenomenology of Inclusive Reactions" ANL/HEP 7134, "Inclusive Experiments a Second Glance" ANL/HEP, 7148; D. Horn, Physics Reports, to be published, W. Frazer et al., Rev. Mod. Physics, to be published. L. Van Hove, Physica Report 1c, 347 (1971).

[2] J. Benecke, T. T. Chou, C. N. Yang and E. Yen, Phys. Rev. 188, 2159 (1969).

[3] R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969).

[4] Scaling properties in the multiperipheral model have been reviewed by S. Fubini, Scottish Universities Summer School (1963). The reader is refered to the original papers or to the review articles of Ref. 1 for a discussion of the theoretical motivations for scaling.

[5] A. H. Mueller, Phys. Rev. D2, 2963 (1970) and Coral Gables Conference (1971), Gordon and Breach, N. Y.

[6] We denote by AB(C) and inclusive reaction where particles A and B collide producing the observed particle C together with anything else.

[7] We denote by s_j the Mandelstam variable corresponding to channel j.

[8] The rapidity difference between A and B increases as Log s. Typically it goes from 4 to 8 between present accelerator energies and ISR energies.

[9] J. Ellis et al., Phys. Letters 35B, 227 (1971).

[10] Chung I. Tan, Phys. Rev. D4, 2412 (1971); J. C. Polkinghorne, Nuovo Cimento 7A, 555 (1972); H. P. Stapp, UCRL 20623 (1971).

[11] H. M. Chan et al., Phys. Rev. Letters 26, 672 (1971).

[12] We of course neglect the energy behavior of the diffractive contribution proper which is responsible for the rise of the K^+p total cross section. We assume that the cross section can be decomposed into a constant asymptotic component and a decreasing Regge component, clearly separated out for instance in the difference between the K^-p and K^+p total cross sections.

[13] H. D. I. Abarbanel, Phys. Letters 34B, 69 (1971).

[14] P. G. O. Freund, Phys. Rev. Letters, 20, 235 (1969); H. Harari, Phys. Rev. Letters 20, 1385 (1969).

[15] H. M. Chan and P. Hoyer, Phys. Letters 36, B 79 (1971); M. B. Einhorn et al., Phys. Letters 37B, 292 (1971); M. Kugler et al., Phys. Letters 38B, 423 (1972).

[16] S. H. H. Tye and G. Veneziano, Phys. Letters 38B, 30 (1972).

[17] R. S. Panvini et al., Phys. Rev. Letters 27, 68 (1971). I am grateful to R. Panvini for many informative discussions.

[18] L. G. Ratner et al., Phys. Rev. Letters 27, 68 (1971); and contribution to the APS Rochester meeting (1971).

[19] As previously stressed what matters is the rapidity interval between A and C. If a pion has the same rapidity as the incident proton its momentum will typically be three times less, with a mean value for q_T of 300 MeV/c its average x value will be about 0.3. One may also see the fragmentation pion as the decay product of an hadronic state obtained through excitation of the initial proton. A 2 GeV state will give pion with typical x value 0.15. We shall, therefore, refer to the $0.1 < |x| < 0.4$ region as the typical fragmentation region at present machine energy. This is where we may probe scaling best. If this applies to pions, the typical fragmentation region is of course displaced toward $|x| = 1$ for heavier secondaries. The same rapidity then corresponds to larger momentum values.

[20] A. Bertin et al., Phys. Letters 38B, 260 (1971).

[21] T. V. Beaupre et al., Phys. Letters 37B, 432 (1971). See also W. D. Shepard et al., Phys. Rev. Letters 27, 164 (1971).

For recent reviews see: E. L. Berger, Helsinki Conference (1971); and T. Ludlam, Invited talk at the APS meeting, Cambridge (1971), Yale preprint (1972).

[22] C. E. de Tar, et al., Phys. Rev. Letters 26, 675 (1971); H. D. I. Abarbanel, et al., Phys. Rev. Letters 26, 937 (1971).

For a phenomenological application see S. Ellis and A. Sanda, NAL preprint (1972).

[23] The same applies to forward inclusive charge exchange reactions. However, the leading RRP term now gives a scaled distribution for the leading particle effect when the RRR term, also noticeable at lower energy, gives a non scaling contribution.

[24] M. Jacob and R. Slansky, Phys. Letters 37B, 408 (1971), and Phys. Rev., to be published (1972).

[25] R. W. Hwa and C. Lam, Phys. Rev. Letters 27, 1098 (1971); R. K. Adair, Phys. Rev. 172, 1370 (1968) and Yale preprint (1972).

[26] The two prong cross section further includes quasi two body ρ and Δ production, which part of the time, gives a slow center of mass π^+. Such cross sections decrease rapidly with energy (isospin exchange) and help maintaining a pseudoscaling property.

[27] Aachen-Berlin-CERN-London-Vienna and Brussels CERN collaboration, Nucl. Phys. B 30, 381 (1971).

[28] E. Predazzi and G. Veneziano, Lettere al Nuovo Cimento 2, 749 (1971); S. H. H. Tye and G. Veneziano, Phys. Letters 35B, 355 (1971); M. Le Bellac, Lettere al Nuovo Cimento 2, 937 (1971); L. Caneschi, CERN preprint (1971).

[29] The leading particle effect is particularly strong and results from the fact that an important fraction of the inelastic cross section corresponds to single excitation, one of the incident particles recoiling on an excited

one. This contributes also to the so called weak inelasticity of production processes.

30 A counter example is offered by L. Von Lindern et al., Phys. Rev. Letters 27, 1745 (1971). This cannot yet be held as a strong case against scaling. Non uniform individual rapidity distributions could build a spurious peak at $x = 0$ since a fluctuation maximum in a rapidity distribution tends to define the energy of the reaction. See A. Krzywicki et al., Caltech preprint (1972).

31 The rise of the inclusive meson distributions at $x = 0$ by a factor two between present machine energy and NAL energy is predicted in the framework of fragmentation models[24]. Model calculations give a much smaller rise (an approximate scaling) through the ISR range of energy. Available results are presented on Fig. 6 which combines data from G. Barbiellini et al., CERN preprint (1972) and G. Neuhofer et al., Phys. Letters 38B, 51 (1972). The ISR data may, however, indicate scaling within the ISR energy range: British-Scandinavian collaboration, Oxford Conference (1972). (Private communication).

32 E. Feinberg, Physics Reports to be published (1972). This paper contains a detailed review of the development of these ideas through Cosmic Ray research.

33 The constant K^-p cross section and the slowly rising K^+p cross section show that the diffractive part, common to both, rises with energy and

compensates the decreasing Regge contribution relevant for K^-p. However, when considered over the whole 10-60 GeV interval, the K^+p total cross section varies much less than the K^-p one. This is the empirical property which we will use here. The same remark applies to the pp and $\bar{p}p$ total cross section.

[34] The empirical fact that pp($\bar{p}$) does not scale does not a priori violate the rule since pp($\bar{p}$) could scale after a belated threshold effect. This is what one would also conclude from the results of Ref. 16.

[35] E. L. Berger, B. Oh and G. A. Smith, ANL preprint 7132 (1971).

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[37] D. Gordon and G. Veneziano, Phys. Rev. D3, 2116 (1971); G. Veneziano, Lettere al Nuovo Cimento 1, 681 (1971).

[38] G. Veneziano, Proceedings of the International Conference on Duality and Symmetry in Hadron Physics, Tel Aviv (1971). I am grateful to G. Veneziano and D. Gordon for several informative discussions.

[39] This analysis was done in collaboration with D. Gordon. One can classify seven new components with different exoticity conditions from

terms obtained from D and DB interference. Some of the terms obtained from D alone scale.

40 We use the terminology of Ref. 24. However, most of the listed properties are common to fragmentation models with diffractive (energy independent) excitation of the beam and target particle. R. W. Hwa, Davis Conference (1972), R. K. Adair, Oxford Conference (1972).

41 The yield of relatively low mass novae strongly depends on their quantum numbers. Since the low mass novae control the inclusive distributions at large rapidity or their general shape at present machine energy, prominent and predictable effects follow. Comparing $\pi^-p(\pi^+)$ to $\pi^-p(\pi^-)$ one expects more π^+ than π^- (almost twice as many on the proton fragmentation side) when more π^- than π^+ are expected on the π^- fragmentation side (about twice as many). A cross over is then predicted around $x = -0.1$. This is a prominent experimental fact.

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43 Z. Koba, H. B. Nielsen and P. Olesen, Phys. Letters 38B, 75 (1972); P. Chliapnikov et al., CERN preprint (1972).

44 E. L. Berger, M. Jacob and R. Slansky, NAL preprint (1972); E. L. Berger, B. Oh and G. A. Smith, ANL preprint (1972); C. Quigg,

M. Wang and C. N. Yang, Stony Brook preprint (1971); R. W. Hwa, Oregon preprint (1972).

[45] H. D. I. Abarbanel, Phys. Rev. D3, 2227 (1971).

FIGURE CAPTIONS

Figure 1 Asymptotic rapidity behavior showing a central plateau and edges with fixed shapes associated to projectile and target fragmentation. Increasing the energy the plateau widens as Log s.

Figure 2

2a Diagramatic representation of the inclusive cross section AB(C). The s_{AB} limits in the two amplitude attached to the exclusive cross sections which are summed over has to be taken with the positive and negative $i\varepsilon$ definition respectively.

2b Regge approximation to the $AB\bar{C}$ discontinuity in the fragmentation limit (the AC rapidity interval is large enough).

2c Regge approximation to the $AB\bar{C}$ discontinuity in the pionization limit (both the AC and BC rapidity intervals are large enough).

Figure 3 A comparison between pion inclusive distributions at 30 GeV/c and at ISR energies.

Figure 4 π^+/π^- and $\bar{p}/p$ ratios at present machine energy and ISR energy.

Figure 5

5a Inclusive distributions for $\pi^+ p(\pi^-)$ at 8 and 16 GeV/c.

5b Inclusive distributions for $\pi^+ p(\pi^+)$ at 8 and 16 GeV/c, Ref. 21.

Figure 6 90° yield for π^0 and charged pions at ISR and accelerator energies.

Figure 7 The triple Regge limit

a) Amplitude

b) Cross section

c) Regge approximation to the cross section

Figure 8 Scattering and production amplitudes

a) Dual scattering V, B and D terms

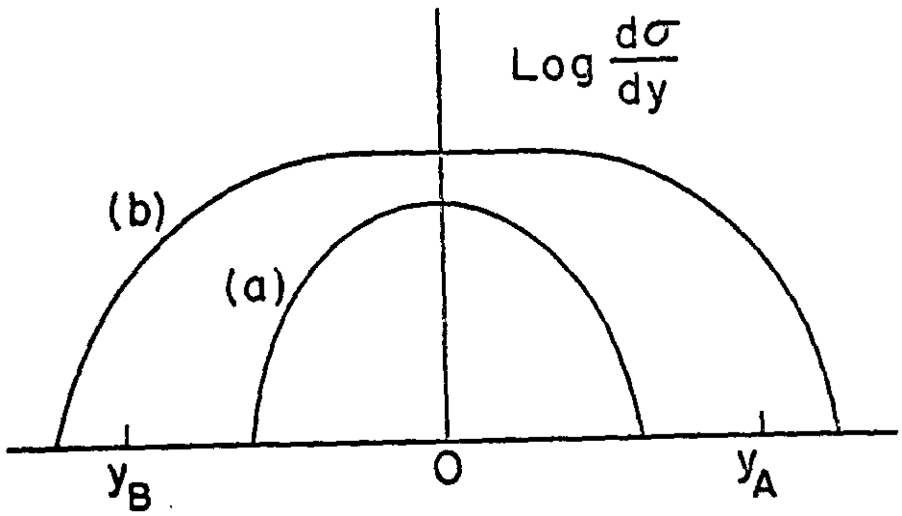
b) Dual production terms

Figure 9

9a Contributions to the discontinuity of the six point function obtained from the three V terms.

9b Production amplitude and its contribution to the discontinuity (5 component).

Figure 10 The $pp(K^-)$ and $pp(\bar{p})$ distribution in the fragmentation region as a function of energy.



pp(π-) (a) 20 GeV/c

(b) ISR energy

Figure I

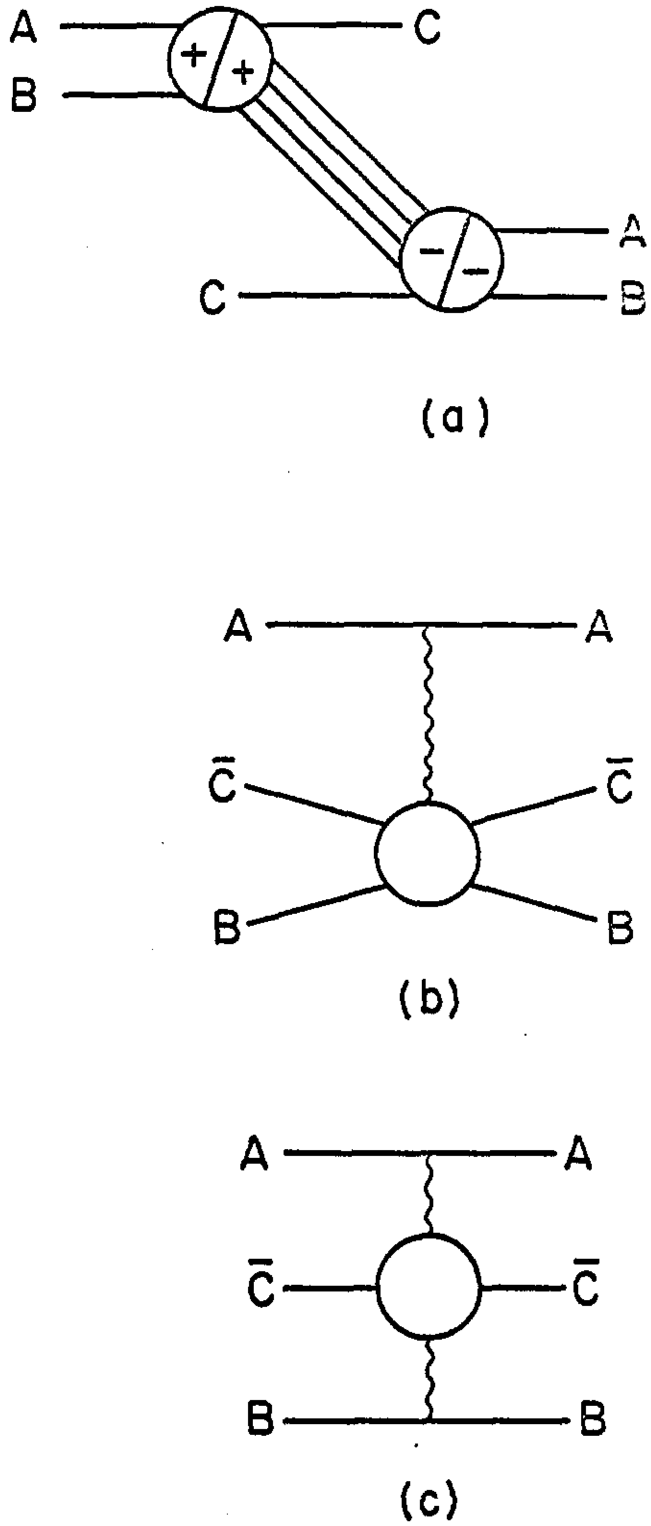
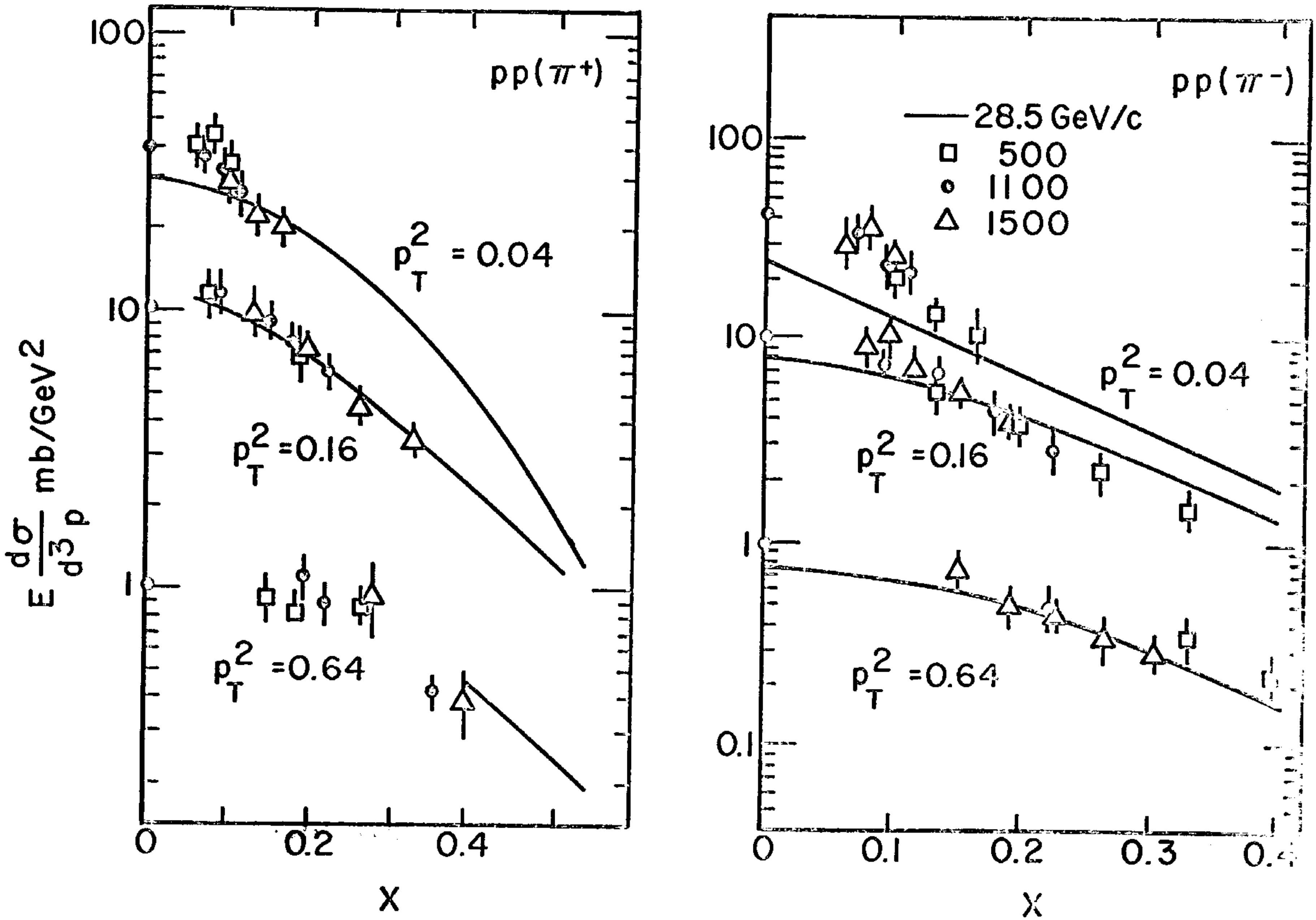


Figure 2



$E \frac{d\sigma}{d^3p}$ mb/GeV^2
100
10
1
0.1
$pp(\pi^+)$
$pp(\pi^-)$
28.5 GeV/c
500
1100
1500
$p_T^2 = 0.04$
$p_T^2 = 0.16$
$p_T^2 = 0.64$
0
0.1
0.2
0.3
0.4
X

Figure 3

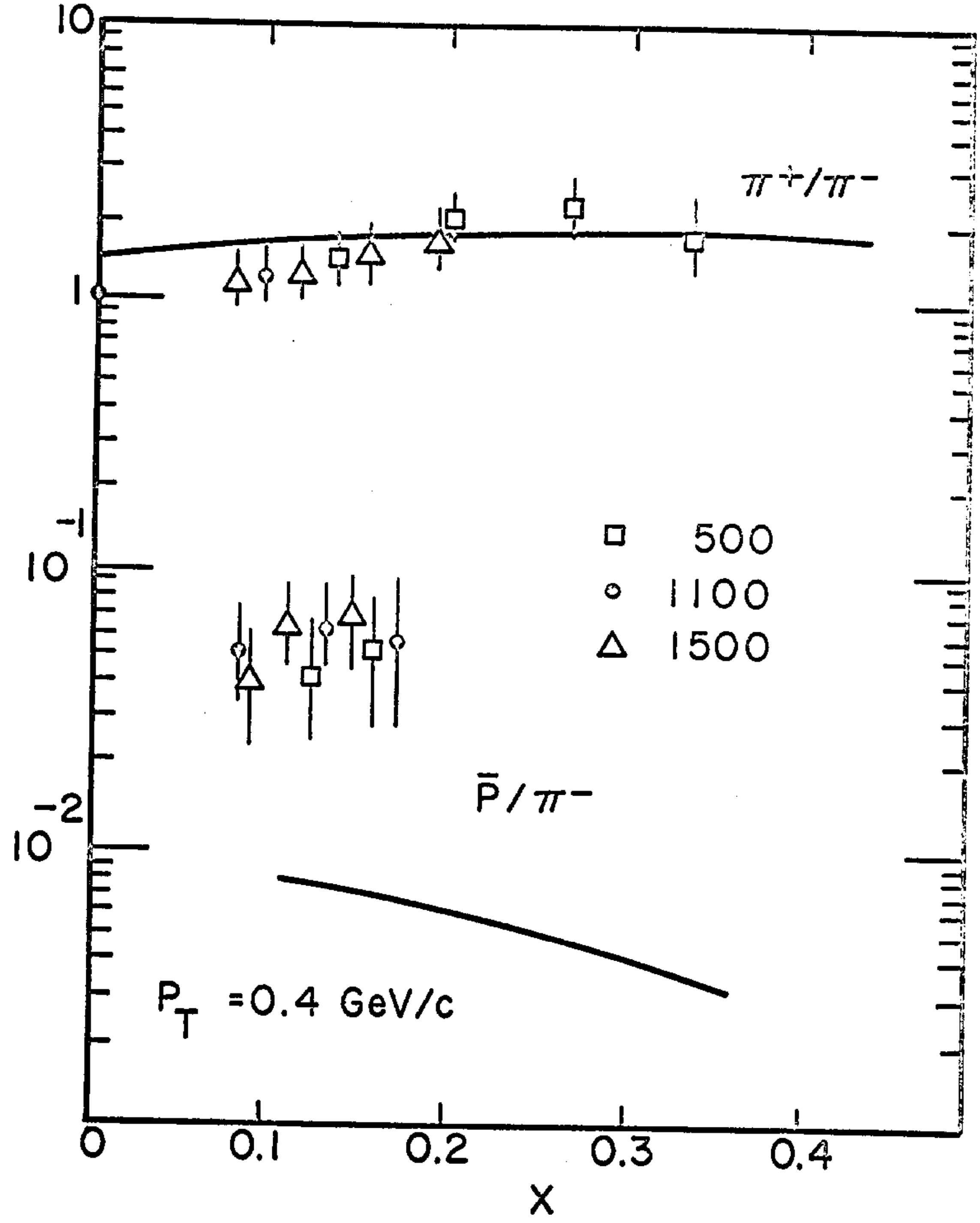


Figure 4

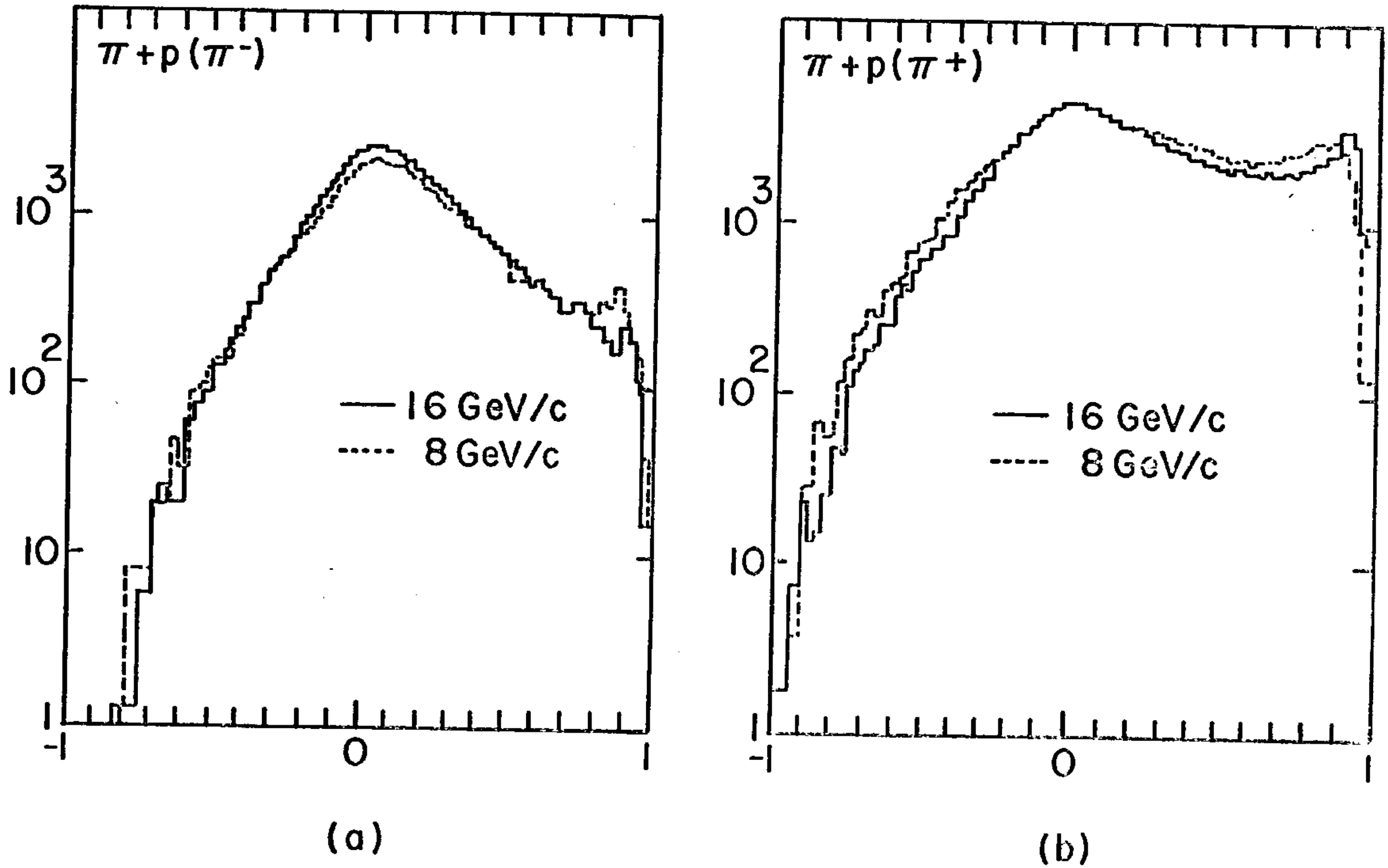

$\pi + p(\pi^-)$
16 GeV/c
8 GeV/c
10^3
10^2
10
1
-1
0
1
(a)
$\pi + p(\pi^+)$
16 GeV/c
8 GeV/c
10^3
10^2
10
1
-1
0
1
(b)

Figure 5

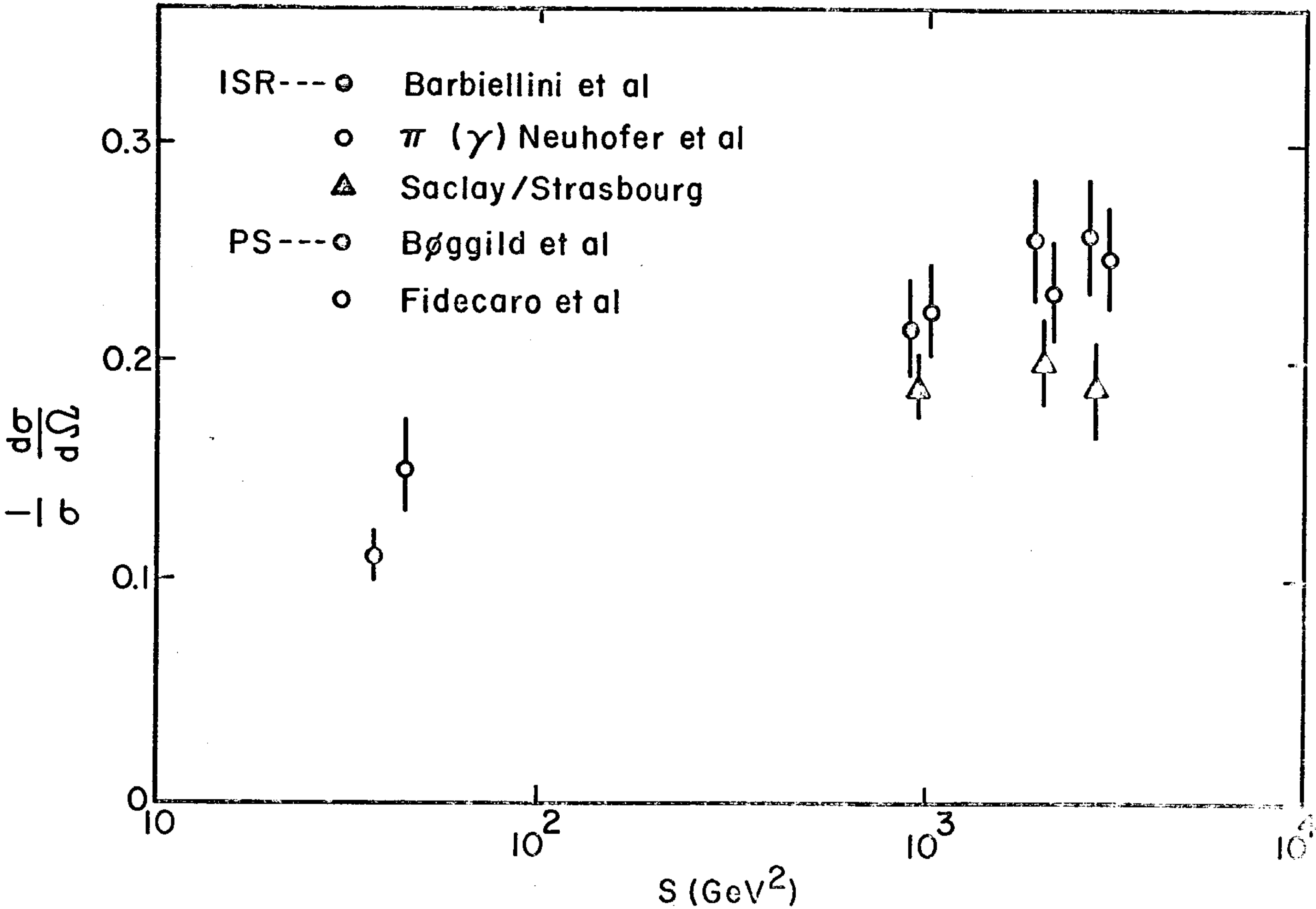

ISR---
Barbiellini et al
π (γ) Neuhofer et al
Saclay/Strasbourg
PS---
Bøggild et al
Fidecaro et al
0.3
0.2
0.1
0
10
10^2
10^3
10^4
$-\frac{1}{\sigma}\frac{d\sigma}{d\Omega}$
S (GeV^2)

Figure 6

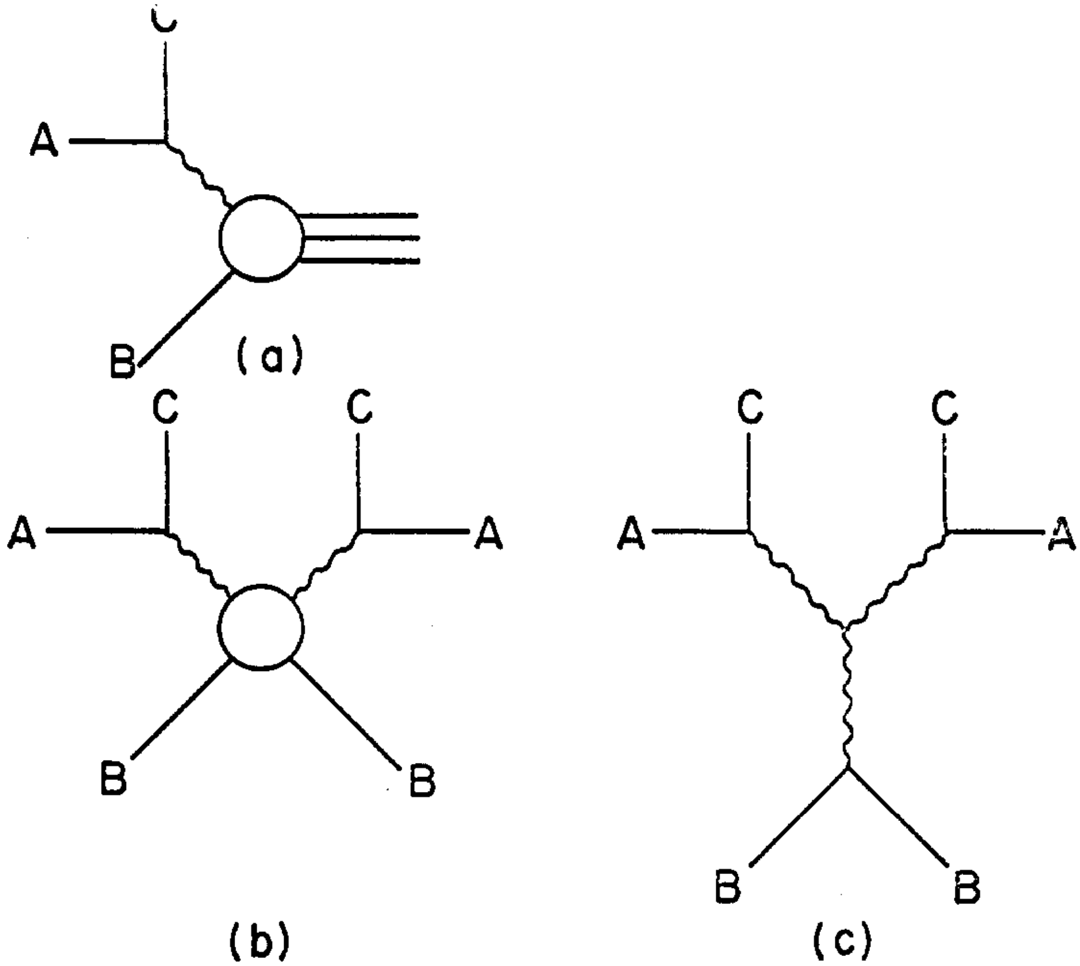


Figure 7

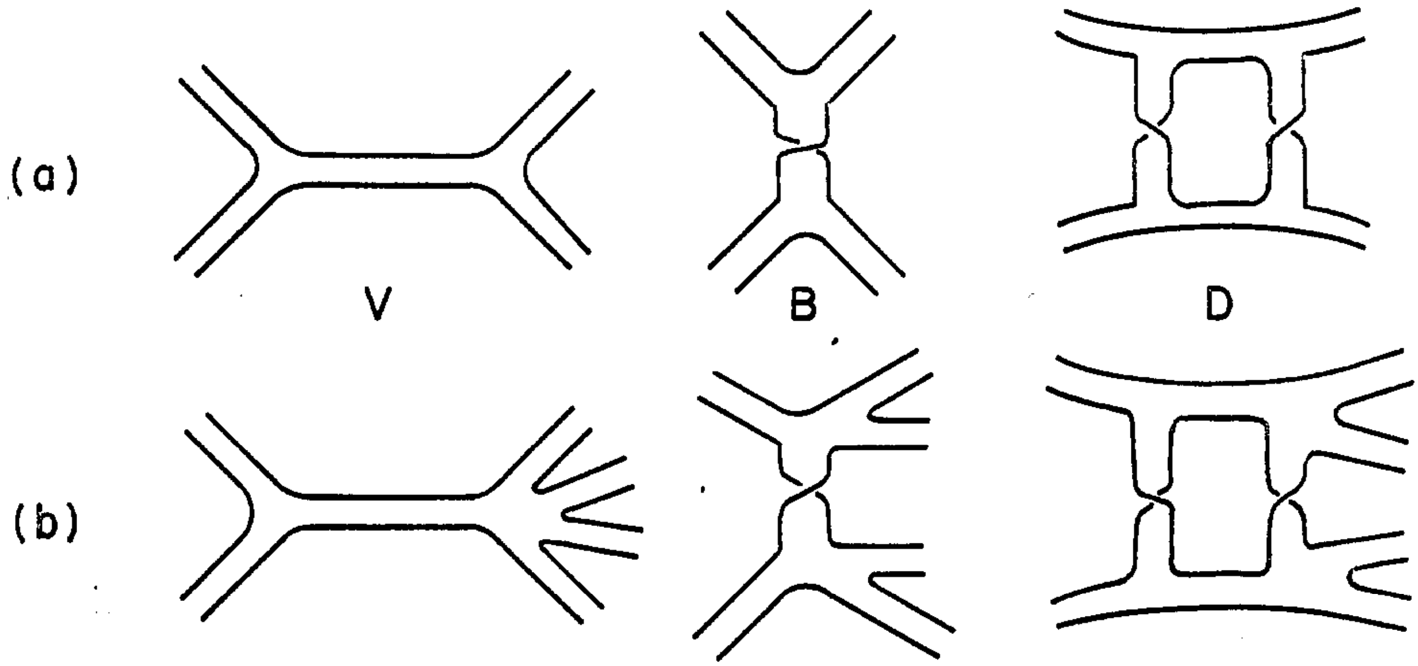


Figure 8

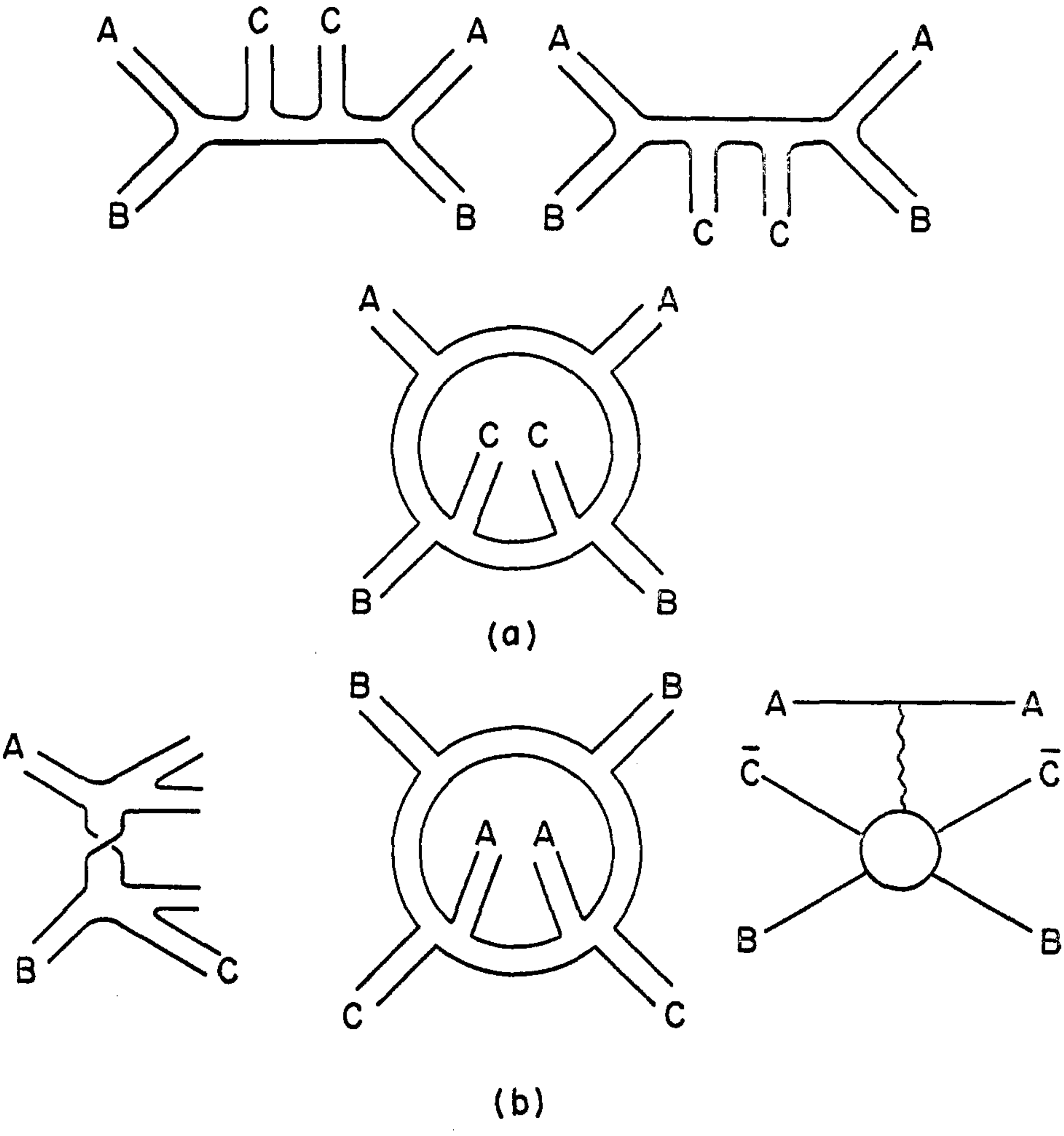


Figure 9

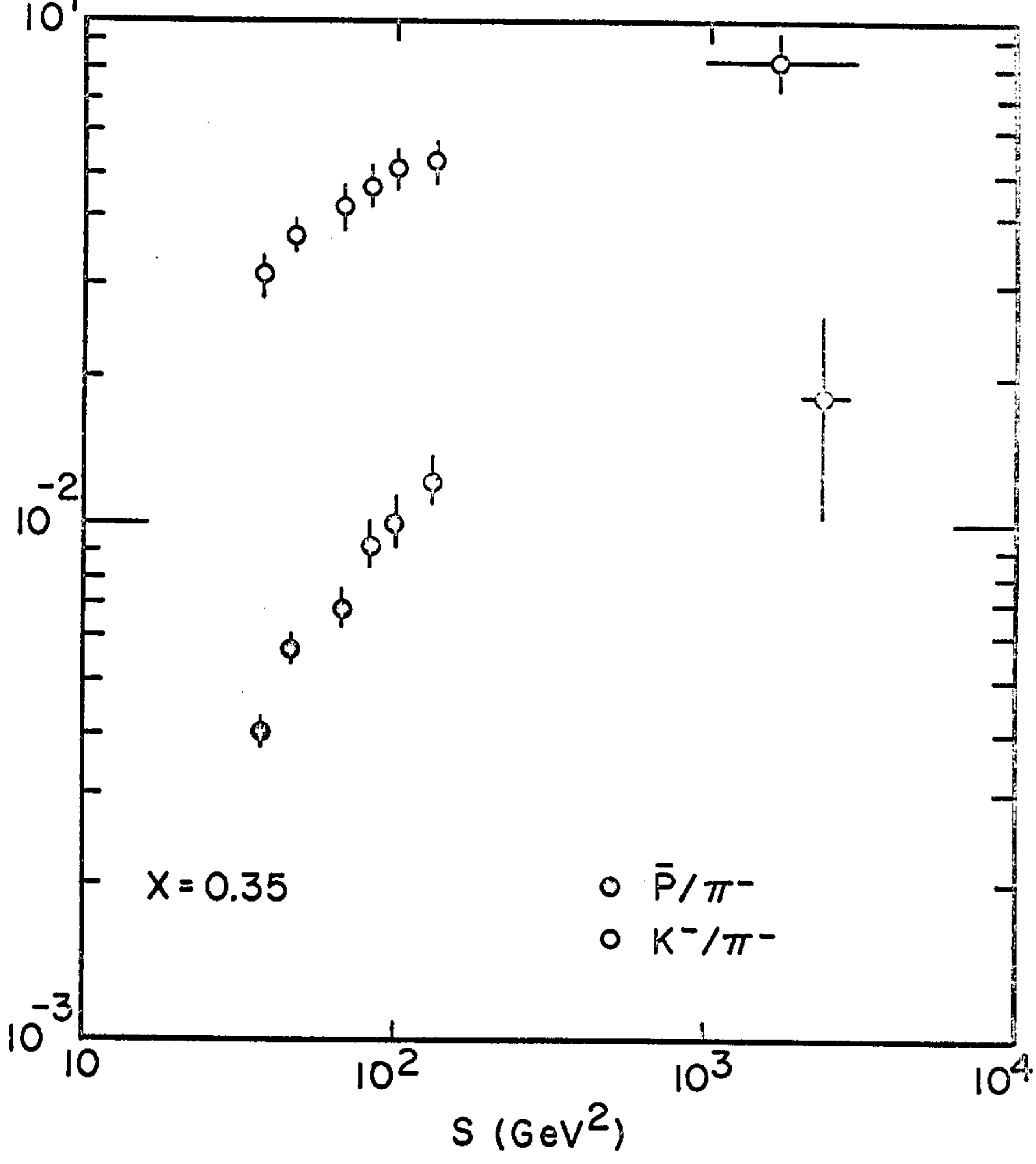


Figure 10